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ON THE DETERMINATION OF THE TAKE-OFF CHARACTERISTICS OF A SEAPLANE

By A. Perelmuter

Central Aero-Hydrodynamical Institute

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INTRODUCTION

The theoretical solution of the problem of a planing body resolves itself into the determination of the magnitude and direction of the velocity at each point of the flow. Having determined these basic elements of the motion it is then not difficult to obtain the forces involved in the planing motion.

The solution of the problem in its most general form with the aid of the hydrodynamical equations, at the present state of our knowledge offers very great mathematical difficulties. For this reason it is usual to simplify the problem by considering only the motion of a flat plate of infinite span (plane or two-dimensional flow). An ideal fluid is assumed, corrections for the viscosity being introduced after the two-dimensional flow has been calculated. This assumption is well founded on the boundary-layer theory of Prandtl according to which the viscosity exerts an effect only within the boundary layer, i.e., the thin layer next to the walls.

According to the method employed for the solution of the problem, the work that has been done by the various authors may be grouped as follows:

1. The work of M. I. Gurevitch and A. R. Yanpolski under the supervision of S. A. Chapligin (reference 1).

The hydrodynamic equations as integrated by the method of Kirchhoff and Joukowski serve as the starting point in the work of these authors. The plate is assumed to be flat and of infinite span and the fluid as ideal and weightless. Expressions are obtained for the pressures on the wetted

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length, and the position of the center of gravity as a function of the initial parameters. Comparison with test results shows that the pressures theoretically obtained exceed the pressures obtained by experiment, being three to four times as great. The qualitative picture of the pressure distribution, however, approaches the actual one very closely.

2. The work of Professors L. N. Sretenski and G. Y. Pavlonko (reference 2).

The flow is assumed to be two-dimensional, the fluid heavy and ideal. To avoid indeterminacy in the solution, dissipative forces are introduced which are made to vanish at the end, assuming the coefficient of viscosity to be near zero. It should be mentioned that the solution of L. N. Sretenski, in contrast to that of Pavlenko, was obtained by strict, mathematical methods. On account of the assumptions made on the nature of the phenomena, however, there is no agreement with experiment.

3. The work of H. Wagner (reference 3).

The work of Herbert Wagner must be considered as the most complete on this subject, its fundamental value consisting in the application of the methods of wing theory to the problem of planing. The results obtained by Wagner for various planing surfaces are in satisfactory agreement with the experimental results. As in the previous works mentioned, the fluid is assumed as ideal.

4. Work of N. A. Sokolov (reference 4).

This work presents a combined theoretical and experimental solution of the planing problem. Theoretical formulas corrected by empirical coefficients are found. The formulas give extremely good agreement with test results. Notwithstanding the fact that the problem has been idealized in the theoretical solution, the investigation is important for the reason that it gives a qualitative picture of the phenomenon and determines the nature of the formula for the forces involved.

The present paper presents an attempt to coordinate the available theoretical and experimental data on planing surfaces so as to develop an approximate analytical method for the determination of the water resistance of a seaplane without any preliminary towing tests in the tank.

NOTATION EMPLOYED

A, load on water, kg.

V, velocity of seaplane, m/s.

W, water resistance, kg.

W_R, frictional resistance, kg.

S, wetted area of seaplane bottom, m².

b, wetted length, m.

b_o, length immersed beneath undisturbed water surface, m.

λ, aspect ratio of wetted surface. $\frac{l^2}{S}$

a, height of wave, m.

F, Froude number.

C_B, load coefficient.

l_K, distance between steps.

L, distance of spray origin from step, m.

M_h, hydrodynamic moment, kg m.

M_1, hydrodynamic moment contributed by nose portion, kg m.

M_2, hydrodynamic moment contributed by second step, kg m.

M_T, moment due to thrust of propellers, kg m.

M_A, moment of load on water, kg m.

M_{BH}, moment contributed by tail surfaces and after portion of seaplane, kg m.

C_f, coefficient of frictional resistance.

α°, angle of attack of bottom with respect to keel line.

β°, angle of V bottom.

G, weight of airplane, kg.

h_p, draft of step, m.

l, beam at step, m.

ρ, density of fluid. 102

g, acceleration of gravity.

p, normal pressure at given point of planing surface.

φ, potential of flow.

t, time.

q, velocity of fluid at given point.

THE FUNDAMENTAL STAGES IN THE TAKE-OFF OF A SEAPLANE

According to our present views on the motion of a viscous fluid, the following picture may be given of the action of the fluid on the surface of the planing body. The reaction of the water on the planing body is the resultant of a system of normal and tangential stresses on the wetted surface. The tangential stresses arise from the property of viscosity and are determined by the motion of the fluid within the thin boundary layer adjacent to the surface of the body. Everywhere outside this layer the viscosity may be neglected and the fluid considered as ideal.

The resultant of the system of tangential stresses is called the frictional resistance. The normal pressures are transmitted through the boundary layer without change and are therefore determined by the motion of the fluid considered as ideal, in particular, as a potential flow. To compute these pressures the Lagrange integral relation may be used, namely,

$$p = \rho \left(\frac{\partial\varphi}{\partial t} - \frac{q^2}{2} - gz\right)$$

The last term in the above formula represents the hydrostatic pressures determined independently of the velocity of the flow. The first two terms represent the "hydrodynamic pressure." Correspondingly, all the forces exerted by the water on the planing surface may be divided into the following three types:

(a) hydrostatic forces

(b) hydrodynamic forces

(c) frictional forces

the first two types being determined by the normal pressure of the fluid, and the third by the tangential stresses between the fluid and the planing surface.

In the process of take-off of the seaplane, each of these types manifests itself to a greater or less extent and thus determines the character of each stage in the take-off. We shall therefore consider the following stages in the take-off: (a) plowing stage; (b) transition or critical stage; (c) planing stage (hydroplaning).

Plowing stage.- This stage extends from the commencement of the take-off up to the attainment of a speed of the order of 0.25 to 0.30 of the get-away speed. A characteristic of this stage is the predominance of the hydrostatic forces which decrease as the speed increases. At the instant of starting, the moment due to the thrust of the propellers and the resistance of the water causes the nose of the hull to "bite" sharply into the water. (It is assumed that the line of action of the thrust passes above the center of gravity of the seaplane.) The trim angle at the nose increases up to the instant when the work due to this moment becomes equal to that of the restoring moment. The waves formed by the seaplane at very small velocities are similar to the waves accompanying the motion of ships and may therefore be divided into two groups: (1) bow waves, and (2) transverse waves. As the speed increases, the bow wave gradually recedes toward the step; the transverse wave disappears and is only observed at the stern. There then appears at the nose, the so-called "blister."*

As the speed keeps on increasing the blister develops into a spray and the position of the center of gravity above the mean water level falls, due to the lowering of the water level about the seaplane.

With increase in the speed, there is an increase in the dynamic pressure at the bottom of the floats and an increase in the lift of the wings. The load on the water,

*The blister is a dome-shaped film of water generally appearing at the nose.

representing the difference between the weight of the airplane and the lift of the wings, decreases. The center of gravity gradually begins to rise. The center of pressure on the bottom shifts toward the nose and increases the trim by the stern.

Transition or critical stage.- With further increase in velocity, the angle of attack and the water resistance increase and reach their maximum value (hump velocity). The center of gravity rises sharply above the water surface. The hull gradually clears the water. The distance between the crests of the transverse waves gradually increases and the waves become more inclined due to the rapid decrease in the draft. Two wave "walls" separate at the rear edge of the step, closing together at the stern and thus forming a "fountain" that recedes from the step as the speed increases. The distance of the spray origin from the edge of the step coincides with the first crest of the transverse wave. The second step lies in the hollow formed by the first and this brings about larger trim angles.

In the "mixed" stage the hydrodynamic forces are of the same order of magnitude as the hydrostatic forces. It should be observed that on account of the maximum (hump) water resistance, which is characteristic of this stage, the latter is the most important during the take-off of a seaplane. We shall therefore, in what follows, begin with this stage.

Planing stage (hydroplaning).- With further increase in the speed, the mean draft of the hull (draft at the step) becomes so slight that the hydrostatic forces may be neglected. The lift is now provided essentially by the dynamic forces. The wetted area decreases, the center of pressure again begins to approach the step at the same time that the angle of attack decreases. The second step now clears the water. The load on the water diminishes approximately in proportion to the square of the velocity at the same time that the water resistance as a rule decreases. Whereas, in the stage described above, the resistance is conditioned by the energy lost in wave formation, in the present stage the friction is the factor of greatest relative importance in producing the resistance. The entire picture of the wave formation changes to a considerable degree. The waves decrease and the spray spreads low over the water.

At very small trim angles instability sometimes appears during the planing, leading in some cases to too early a break-away from the water when the wings have not yet attained sufficient lift for get-away. Longitudinal instability is also observed at times at the instant of break-away of the second step.

CHARACTERISTICS OF THE TAKE-OFF STAGES

For a numerical determination of the limits of each stage of the motion, the nondimensional coefficient

$$C_B = \frac{A}{\frac{\rho}{2} l^2 V^2}$$

may be employed. By comparing airplanes of different types, the following limits for C_B were established:

(a) critical velocity stage, $C_B = 0.1$ to 0.25

(b) hydroplaning stage, $C_B = 0.09$ to 0.04

(c) velocities just before take-off, $C_B \geqq 0.02$

The motion may also be conveniently characterized by the "Froude number" in the following form:

$$F = \frac{V}{\sqrt{g \sqrt[3]{A}}} \lambda$$

where $\lambda = \frac{l^2}{S}$

A = the water displacement in cubic meters

Figure 1 shows the value of F as a function of the ratio: hydrostatic lift/load-on-water for flat plates (tests at CAHI tank and by Sottorf).

On figure 2 are given the curves of trim angle and draft at the step as functions of the velocity for a model float towed on the water at constant load. The tangents drawn to these curves indicate the limits of the various stages. Comparing these results, the following values of F may be used to indicate the limits of the various stages:

1) plowing stage, F up to 0.5

2) transition stage, F from 0.5 to 2.5

3) hydroplaning stage, $F > 2.5$

The coefficients C_B and F sufficiently well characterize the fundamental stages in the take-off of the seaplane.

PLANING OF A FLAT PLATE

Before proceeding to the consideration of the planing of a seaplane float, we shall consider the more simple case, namely, the planing of a flat plate at a constant velocity V. The planing will be denoted as that stage of the motion during which the water wets only the lower surface, breaking away from the edge of the plate. The following characteristic properties of the motion of the water are observed:

(a) a thin stream of water, the so-called "spray," separates at the leading edge of the wetted surface and is thrown off ahead of the plate.

(b) behind the plate, where the side waves meet, there is observed a "fountain" that attains considerable force as the load on the water is increased.

The water will exert the following forces on the planing plate:

(a) the resultant of the normal pressures due to the reaction of the fluid and acting perpendicular to the plate.

(b) the resultant of the frictional forces due to the viscosity and acting along the surface of the plate (fig. 3).

Evidently the resistance of the plate will be given by the following formula:*

$$W = A \tan \alpha + C_f \cos \alpha \, V_{mean}^2 \frac{\rho}{2} S \qquad (1)$$

Let us consider the second term in the above formula, expressing the effect of the viscosity of the fluid. In making the hydrodynamical computation the magnitudes to be determined are:

$$C_f, \quad S = \frac{l^2}{\lambda}, \quad V_{mean}$$

For computing the frictional coefficient C_f, we shall consider as applicable the formula of Prandtl (for the turbulent stage) that was proposed by him for the computation of the friction of a completely immersed plate moving with constant velocity** in its plane:

$$C_f = \frac{0.072}{\sqrt[5]{Re}}$$

This formula applied to the computation of the resistance gives good agreement with test results (reference 5). The linear dimension in the Reynolds formula will be taken as the immersed length computed by the formula:

$$b_o = \frac{b}{\sqrt{1 + \lambda}}, \quad Re = \frac{V_{mean} \, b_o}{\nu}$$

The aspect ratio of the immersed area will be obtained from the curves, figures 4, 5, and 6. These figures present the graphical solution of the equation of lift as a function of λ.

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*All the formulas for flat plates were taken from the paper by N. A. Sokolov: "On the Hydrodynamic Computation of Floats and Seaplanes," CAHI Report No. 129, 1933; and from the work of the author: "Hydrodynamic Computation for a Flat-Bottomed Seaplane Float," CAHI Technical Note No. 48, 1935.

**In place of the above formula, it would also be possible to use more accurate formulas for the determination of C_f. Bearing in mind, however, on the one hand, that the formula for C_f for a completely immersed plate may only be used with a certain amount of reservation, and on the other, that formula (2) has up to now given good agreement with the tests on planing plates, we consider the formula to be quite satisfactory for our purpose.

$$\frac{C_B}{2\alpha} = \left[\frac{1}{1+\lambda} + \frac{1}{\lambda^2}\frac{1}{F^2}\left(\frac{1}{\sqrt{1+\lambda}} - 0.5\right)\right] \qquad (2)$$

For the case of large aspect ratios $\lambda > 3$, the terms involving gravity may be neglected and the formula for the determination of λ then assumes the following more simple form:

$$\lambda = \frac{2\alpha}{C_B} - 1$$

The mean velocity under the plate we shall define by the formula

$$V_{mean} = V\left(1 - \frac{\alpha\lambda}{1+\lambda}\right)$$

The lift is in this case equal to the load on the water

$$Y = A$$

The moment of the hydrodynamic forces about the step is determined from the equation

$$2\frac{M}{Yl} = \frac{\frac{1}{\lambda}\frac{1}{F^2}\left(\frac{1}{\sqrt{1+\lambda}} - \frac{2}{3}\right) + \frac{\pi}{2}\frac{\lambda}{1+\lambda}}{\frac{1}{F^2}\left(\frac{1}{\sqrt{1+\lambda}} - \frac{1}{2}\right) + \frac{\lambda^2}{1+\lambda}} \qquad (3)$$

The solution of this equation is given graphically on figures 7 and 8.

For the case of large aspect ratios, the formula assumes the extremely simple form:

$$\frac{M}{Y} = \frac{\pi}{4} b$$

The immersion of the rear edge (draft) is determined from the formula taken from the work mentioned above:

$$h = \frac{\alpha l}{\lambda\sqrt{1+\lambda}}$$

PLANING OF A V-BOTTOM PLATE

The forward portion of a seaplane differs considerably from that of a flat plate, due to the V-angle which in general varies along the length. To simplify the computation of the keel angle of the bottom, we shall take as a mean value the inclination to the horizontal at the mean section over a distance of twice the width of the step. It should be observed that for most of the present-day seaplanes this angle is near zero. As far as the V-angle is concerned, we shall give formulas below that take this angle into account for the case of a straight V-bottom. For the curved bottom case, the V-angle may be taken into account using the formulas of Wagner or the equivalent straight V-bottom (fig. 9).*

Obviously, the resistance of the V-shaped bottom is determined by the same formula (1) that applies to the flat bottom. In using the formula, the difference lies only in the definition of the terms giving the frictional resistance. The latter will be larger for the V-bottomed plate than for the flat plate for the same values of α, A, and V, and this increase in the resistance may be taken approximately proportional to the increase in the wetted area.** Thus, we see that: (1) the correction for the increase in the resistance of the V-shaped bottom as compared with the flat bottom, will enter only in the frictional resistance; (2) the correction factor will depend only on the wetted-area aspect ratio.

WAGNER'S METHOD OF COMPUTATION FOR V-BOTTOMS

The planing motion of the body on the surface of the fluid may be pictured as follows: Forward of the body the surface of the water is practically undisturbed. In the

*The V-angle of the equivalent straight bottom is taken as the arithmetical mean of the inner and outer V-angles.

**The frictional resistance depends also on the velocity distribution along the plate and thus, for example, a possible increase or decrease in the spray may increase or decrease the frictional resistance. Calculation shows, however, that this factor is of far less significance than the wetted area.

immediate neighborhood of the body a mass of water rises upward and is deflected downward and to the sides. Part of the water separates and breaks up in the form of a spray.

H. Wagner has shown that it is possible, for the above reason, to draw an analogy between the impact on the water surface of an infinitely long V-shaped plate and planing (reference 6). The analogy will be closer the greater the aspect ratio of the wetted area of the planing body. In this case large accelerations arise in the water and the phenomena of landing and planing become very similar. Making use of the above analogy, Wagner proposes the following formula for the ratio between the lifting forces of the V-bottom and flat-bottom plates, respectively:

$$\frac{A_1}{A} = \eta$$

where $$\eta = 1 - \frac{\beta}{\pi} - 0.15\,\frac{u}{\pi} - \frac{u}{\pi}\log\frac{1}{u}$$

and β is the V-angle, $u = 2\pi/\beta$.

It has already been pointed out above (See the author's work cited in footnote, p. 9) that the lift of a flat planing plate is determined by the formula:

$$A = \frac{2\alpha}{1 + \lambda}\,V^2\,l^2\,\frac{\rho}{2} \tag{4}$$

Therefore, from (3) for a V-bottom plate the expression for the lift force becomes:

$$A_1 = \eta\,\frac{2\alpha}{1 + \lambda}\,V^2\,l^2\,\frac{\rho}{2} \tag{4'}$$

If we consider the motion of a flat-bottom and V-botton plate for the same values of V, l, and α, then it follows from (4') and (4) that in order to obtain the same lift $A = A_1$ for each of the plates, it is necessary to introduce a correction only in the aspect ratio λ_V of the wetted area, as is confirmed by experiment since $\lambda_V < \lambda$, and if this correction is not introduced then (see formula (4')), the lift of the V-bottom plate will be too large.

Thus for $A = A_1$, $V = V_1$, $\alpha = \alpha_1$, $l = l_1$, we obtain:

$$1 = \frac{1 + \lambda_{flat}}{\dfrac{1 + \lambda_V}{\eta}}$$

On figure 11 are shown the results computed by the above formula. The test points of Sottorf for plates with β = 10°, 15°, 24°, 40°, were recomputed for $\beta = 0°$. The corrections for the V-angle were taken from figure 10. It may be seen from figure 11 that the points give a rather wide scattering and for this reason we consider the formula to be applicable only to large aspect ratios where a closer approximation is obtained.

EXPERIMENTAL CORRECTION FORMULA FOR THE V-BOTTOM

In seeking to obtain the correction factor for the V-bottom (increase in wetted area as compared with that of a straight bottom), it was found necessary to make the assumption that η depends not only on the V-angle but also on the angle of attack α. Thus,

$$\eta = f(\alpha)\, f(\beta)$$

By working up the test data of Sottorf, a correction for the V-bottom was found of the form

$$\eta = 434 \sin \alpha \sin^2 \beta \qquad (5)$$

On figure 12, which serves as a basis for formula (5), are given the values of S/l^2 against the angle β for constant values of the angle of attack. (See tests of Sottorf for planing plates.) By dividing each of the values S/l^2 at α = constant by the corresponding value for the flat plate, the relative increase in the wetted area of the V-bottom plate was found under analogous conditions at each α = constant. By then drawing the curve of η/α against β, the analytical expression for the V-bottom correction was obtained in the form of expression (5). Figure 13 gives the computed results for a bottom of angle $\beta = 24°$ (CAHI tank tests). The test points are also indicated. Figures 14 and 15 show curves of computed resistance against speed for V-bottom plates of angles β =

30° and 40° for constant angles of attack and constant draft of the rear edge $h_p = 96$ mm (reference 7). The computation was carried out according to formula (1) with the correction for the V-bottom according to formula (5). The test points also given on the same figures show only a very slight deviation from the computed curves.

Figure 16 shows a computed example for a curved V-bottom plate (CAHI tank tests). The angle β was here taken to be the mean between the inner and outer angles in accordance with the observation made above. The test points show that satisfactory agreement is obtained.

BASIC ASSUMPTIONS MADE

Before proceeding with the explanation of the method of the hydrodynamic computation of the seaplane, we shall enumerate the fundamental assumptions made with regard to the character of the planing phenomenon, which assumptions are necessary for the simplification of the computation.

1. The nonuniform motion of the seaplane in take-off we shall replace by a succession of uniform motions with corresponding constant speeds. This assumption corresponds to the procedure usually adopted in towing tests on hull and float models in the tank.

2. The main step of the seaplane or float appears to be the supporting step at which the entire hydrodynamic lift is produced so that the term $A \tan \alpha$ refers only to the main step. Actually any deviation from this formula shows up in the difference between the angles of attack with respect to the flow at infinity at the first and second steps. Due to the relative unimportance of this deviation, we shall neglect it.

3. In considering the forces acting on the portion of the bottom in the regions of both steps, the following assumptions may be made:

(a) The main step may be considered as an isolated planing plate moving at the given angle of attack.

(b) The geometric position of the second step with respect to the water is determined by the angle of attack and draft of the main step and the position of the second step with respect to the main step.

(c) The actual angle of attack of the second step is equal to that with respect to the undisturbed water surface plus the "downwash" determined by the profile of the wave surface formed by the bow portion of the bottom.

(d) Under certain take-off conditions (at and beyond the critical velocity) the second step is immersed in the fluid and produces a "diving" moment. For computing the moment we shall assume that in this case the loads on the steps are proportional to the squares of the bottom widths at the steps:

$$\frac{A_1}{A_2} = \frac{l_1^2}{l_2^2}$$

It therefore follows that the aspect ratios of the first and second steps are approximately equal:

$$\lambda_1 = \lambda_2$$

(e) The resistance of the second step enters only as a frictional resistance.

4. The motion of the fluid at the second step of the seaplane is very complicated. We do not have, at the present time, any theoretical or experimental data that provide a full explanation of the complicated picture of wave formation beyond the second step of the seaplane. Similarly, we do not know to what extent the velocity of the fluid at the second step differs from the towing velocity of the model.

At the CAHI tank the velocity of the flow was measured at 200 mm behind the edge of the plate with a Prandtl tube. The measurements at 5 and 7 meters per second show an almost complete agreement of the towing velocity of the plate with the velocity of flow behind the plate. There were also measured the velocities under the second step of the planing body (at a distance of 150 mm from the edge). The difference between the towing velocity of the model and the velocity of the fluid reached 3 percent. Not having any more accurate basis with regard to the variation of the velocity of the flow behind the main step, we considered it permissible to assume that the velocity of the fluid at the second step was equal to the speed of motion of the seaplane.

WAVE PROFILE BEHIND MAIN STEP

Not possessing the means for constructing the profile of the disturbed water surface behind the step of the planing V-bottom, we shall limit ourselves to obtaining the profile behind the flat planing bottom.

The construction of the wave profile in the diametral plane behind the planing flat plate may be accomplished with the aid of formulas given by us in a previous work (reference 8). For the case of a two-step hull we shall limit ourselves to the computation of the following coordinates of the profile:

(1) the distance of the spray origin from the edge of the first step $L = l\left(2F - \frac{1}{\lambda}\right)$

where $F = \frac{v}{\sqrt{lg}}$;

(2) the maximum lowering of the wave profile

$a = \frac{2\pi}{3}\frac{l \sin \alpha}{\lambda}$

where a is at a distance $\frac{L + b}{2}$ from the edge of the step;

(3) the draft at the stern $h_p = \frac{\alpha\ b}{\sqrt{1 + \lambda}}$.

Joining the two points of the profile thus obtained by straight lines, we obtain the direction of the flow at the second step.

The above method may be applied only for the case $\lambda > 0.5$. When $\lambda < 0.5$, the following working hypothesis based on experimental observations is recommended, namely, that the fluid breaking away at the main step rises up to the undisturbed water level and is directed along a line forming an angle with the horizontal equal to the angle of attack of the bottom.

METHOD OF COMPUTATION OF THE HYDRODYNAMIC RESISTANCE OF A HYDROPLANE

Let us consider the conditions of equilibrium of the forces acting on the seaplane moving with constant velocity V at a given angle of attack α of the first step.

The projection of all the forces on a vertical gives the conditions determining the load on the water:

$$A = G\left(1 - \frac{V^2}{V_{g\alpha}^2}\right) \qquad (6)$$

The get-away velocity for each given angle of attack α is obtained by the formula

$$V_{g\alpha}^2 = \frac{G}{C_{y\alpha}\ \rho\ S}$$

where S is the area of the wings

$C_{y\alpha}$, the lift coefficient determined from the airplane polar

The projections of all the forces on the horizontal give*

$$m\frac{dV}{dt} = \Phi - Q - W \qquad (7)$$

where Q is the air drag of the airplane

Φ, the propeller thrust

Setting equal to zero the sum of the moments of all forces with respect to the edge of the main step (the term involving the angular acceleration will be neglected), we have:

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*At each constant speed $\Phi = W$.

$$M_h - M_A - M_\Phi - M_{ex} = 0$$

where M_h is the sum of the moments of the hydrodynamic forces acting on the nose portion of the hull and on the second step.

M_{ex} is the external trimming moment contributed by the stern portion of the hull or floats at the critical velocity and the moment due to the elevator in hydroplaning.

Curves of resistance and trimming moments of the nose portion of the hull or float are drawn as functions of the speed for constant trim angles, the lift action of the wings being taken care of by formula (6). The trim angles are chosen to lie within the practical range in seaplane take-off.

The moment contributed by the load on the water and that due to the propeller thrust are found and the difference $M_h - M_A - M_\Phi$ is formed. M_{ex} is obtained as a function of V for constant angles of attack.

By comparing the curve of external moments against V with the curve of frictional resistance W against V for the same constant trim, we can draw the curve of resistance of the seaplane in take-off for a predetermined M_{ex}. It is also possible to draw the resistance curve in take-off for $M_{ex} = 0$ (free to trim). For this purpose it is necessary, on the curve of external moments, to take the points of intersection of the M_{ex} curves with the velocity axis at constant values of α.

EXAMPLES OF COMPUTATION

Twin-float seaplane "Avro"

We shall determine analytically the resistance curve of the floats at constant towing velocities. The thrusts will be taken as equal to the resistance.

We shall assume that the mutual interference between the floats is slight so that it can be neglected and therefore the resistance of the pair of floats is twice the resistance of a single float.

Fundamental Design Parameters Needed for the Computation

Weight in flight $G = 1.12$ tons

Width of bottom along step of float $l = 0.607$ m

Abscissa of center of gravity from the step along a horizontal $x = 0.495$ m*

The ordinate of the propeller axis (along a perpendicular to the horizontal) $y = 2$ m

The inclination of the mean section to the horizontal $= 0°$.

The V-angle β of the bottom $= 5°$

The order of the computation is the following:

1. A is determined as a function of V for each angle of attack by formula (6).

2. For each speed and each angle of attack, we find; $C_B = \dfrac{A}{\frac{\rho}{2} l^2 V^2}$ after which, from a knowledge of $C_B/2\alpha$ and lg/V^2 (1), we find λ for the flat bottom.

3. A correction for the V-angle is introduced (formula 5):

$$S_V = S_{flat} + \eta S_V \qquad S_{fl}(1+\eta)$$

4. b_o is determined from the formula

$$b_o = \frac{b}{\sqrt{1 + \lambda}} .$$

where b and λ are corrected for V-angle.

5. Having computed Re and C_f, resistance curves of the floats as functions of the speed are drawn at $\alpha = 4°, 6°, 8°, 10°$ (fig. 17).

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*The horizontal was taken to be the longitudinal axis of the seaplane lying in the plane tangent to the keel line of the float at the step.

6. The sum of the moments of the hydrodynamic forces is determined from figures 7 and 8, having first found the coefficients lg/V^2 and λ.

7. Having computed the moment due to the thrust $M_\Phi = 2y\ W \cos \alpha$ and the moment due to the load $M_A = Ax \cos \alpha$, we form the algebraic sum of the three moments found and obtain the moment M_{ex}. (See fig. 18.)

We can now without difficulty draw the curve of water resistance of the floats at constant speeds for previously given probable external moments (fig. 19).

Comparison of the computed resistance curve with the experimental curve obtained from the CAHI tank tests (reference 9) on the full-scale "Avro" seaplane shows that the method described gives satisfactory results for single-step floats.

We shall now make a similar computation for the seaplane Stal-3.

Data

Weight in flight $G = 2.78$ tons.

Broadth of step of float $l = 0.95$ m.

Abscissa of the center of gravity along horizontal from step $x = 0.36$ m.

Ordinate of a propeller axis $y = 2.397$ m.

Inclination of mean section of working area to horizontal = 1°.

V-angle at step $\beta = 21^\circ$ (averaged).

Carrying out the computation according to the procedure described above, we obtain the computed curves (figs. 20 and 21). Comparison of the curve obtained (fig. 20) with the curve obtained experimentally in the CAHI tank and reduced to full scale by cubing the model scale, shows that a satisfactory agreement was obtained. The computed curve lies entirely below the test curve, the difference between them amounting to not more than 11 percent.

Tables showing the computed characteristics are given at the end of the paper.

Two-Stepped Hull

As an example of the obtaining of the hydrodynamic characteristics of a flying boat, we present a computation carried out for a seaplane whose hull has the lines of the N.A.C.A. model No. 11 (Reference 10).

D a t a

Weight in flight $G = 6.8$ tons.

Breadth of main step $l = 2.58$ m.

Mean breadth of second step $l_1 = 1.2$ m (area at working portion divided by the length).

Abscissa of center of gravity along a tangent to the keel from step to nose $x = 0.93$ m.

V-angle of bottom at main step $\beta = 22.5^\circ$.

V-angle of bottom at second step $\beta = 22.5^\circ$.

Inclination of mean section to base line (at working area) $= 0^\circ$.

Distance between steps $l_k = 4.25$ m.

As we pointed out above in the general balance of the resistances, that of the second step enters only as frictional resistance.

The resistance and hydrodynamic forces must be computed in the case where the fluid flows up to the second step. The contact of the water with the second step was taken into account by drawing the corresponding wave form in the plane of symmetry beyond the step as in the previous examples. The wetted area of the second step was determined from the condition that the aspect ratios at the main and second steps were equal.

We shall compute the resistance curves for the hull for angles of attack of 3°, 5°, 7°, and 9°. The wetted area is determined from the condition

$$S = \frac{l^2}{\lambda}$$

where λ is found from figures 4, 5, and 6. Introducing the correction for the V-angle according to formula (5) and summing up the resistances of both steps,

$$W = W_1 + W_2$$

we obtain the resistance curves of the first and second steps at α = constant (fig. 22).

On figure 23 are shown the curves of hydrodynamic moments for $\alpha = 3^\circ$, 5°, 7°, and 9°. The computed tables are given at the end of this paper.

We shall now draw the resistance curves for the given angles of attack taken from the test. Figure 24 shows the computed resistance curve at get-away and the experimental curve reduced to full scale, according to the Froude law (reference 10).* The agreement between the two curves in the range of pre-get-away velocities may be improved if the following considerations are taken into account.

In towing the model at constant load in the tank, there is observed only an extremely small change in the amount of immersion of the step. The immersion of the step remains practically equal to a certain constant magnitude and, therefore, assuming a constant immersion, we are led to the conclusion that the aspect ratios and hence the wetted areas at α = constant, will not change. Having made the corresponding computation, we obtain an approximation which is in satisfactory agreement with the computed curve obtained at the towing tank at Langley Field (figs. 25 and 26). On figure 27 are shown samples of resistance calculations for a load $A = 1945$ kg and at an angle of attack $\alpha = 5^\circ$ with and without rising of the step. Figure 28 also shows a sample of calculation for a model to 5.97 scale. On the same figure is also shown the test curve obtained in the tank.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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*The computation was carried out in the N.A.C.A. tank.

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$5\frac{1}{2}^{\circ}$ [illegible]
$+1^{\circ}$ [illegible]
$6\frac{1}{2}$ [illegible]
~~[illegible]~~ 3.3% b

TABLE I

Hydrodynamic Characteristics of Model NACA-11 Hull

at $\alpha = 5^{\circ}$; $V_{get\text{-}away} = 31.3$ m/s

V	A	C_B	λ	S with correction for V angle	b with V angle	W_R	A tan α	W_1	W_2	W	W with constant immersion
8	6420	0.295	0.4	26.1	10.1	170	562	732	43.6	775.6	775.6
10	6110	.18	.45	23.26	9.0	230	535	765	59	824	824
12	5800	.119	.64	16.2	6.28	240	507	747	35.6	782.6	783
16	5020	.0579	1.87	5.58	2.18	172	439	611	--	611	611
20	4030	.0298	4.9	2.138	.826	119	353	472	--	472	520
24	2800	.01435	11.25	.957	.371	87	245	332	--	332	424
28	1360	.00511	33.4	.312	.121	46.9	119	189	--	189	400

TABLE II

V	A	A_2	A_1	M_1	λ	M_2	M_h
8	6420	1140	5280	22500	0.255	2665	20320
10	6110	1083	5017	22600	.287	2730	20800
12	5800	1030	4770	17950	.461	1900	15500
16	5020	890	4130	8400	--	--	8400
20	4030	712	3318	630	--	--	630
24	2800	497	2303	815	--	--	815
28	1360	240	1120	130	--	--	130

TABLE III

V	8	10	12	16	20	24	28
h_p	-	0.6925	0.464	0.1295	0.0356	0.0115	0.00225
a	-	1.645	1.15	.258	.152	.068	.0222
L	-	1.24	6.06	15.0	19.7	24.03	28.58

TABLE IV

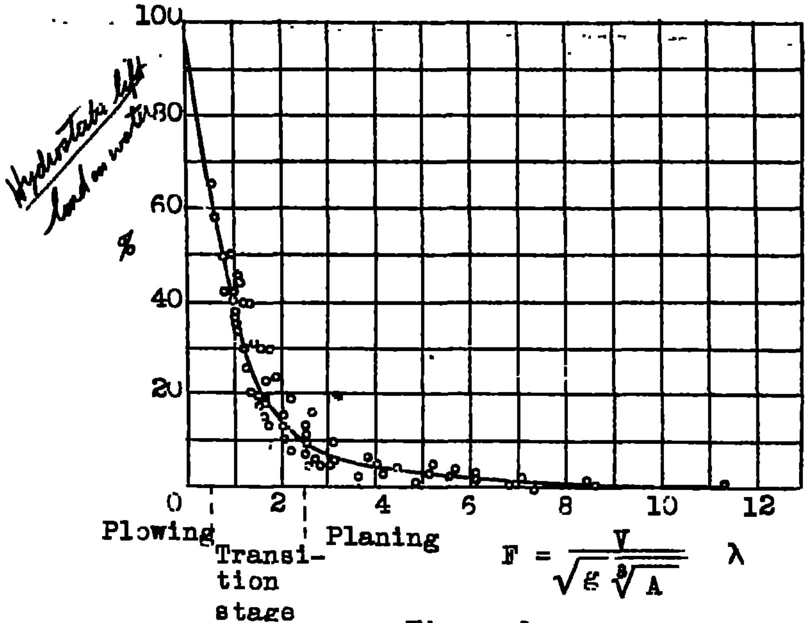
Hydrodynamic Characteristics of Stal-3

at $\alpha = 8^\circ$; $V_{get-away} = 32.9$ m/s

V	A	C_B	S with correction for V angle	λ with V angle	W_R	A tan α	W	W with constant immersion
6	2860	0.81	6.67	0.135	28.2	188	216.2	216.2
8	2610	.444	5.93	.1525	43.1	183	226.1	226.1
10	2520	.274	4.58	.197	52.6	177	229.6	229.6
12	2400	.181	3.08	.293	50.6	168.5	219.1	219.1
14	2260	.125	1.57	.575	41.7	158.5	200.2	200.2
16	2100	.089	.825	1.093	32.7	147.5	180.2	180.2
20	1720	.0467	.323	2.8	22.4	121.0	143.4	160
24	1255	.0235	.148	6.1	19.45	87.5	106.95	123
28	710	.00985	.0589	15.3	13.3	49.9	63.2	81

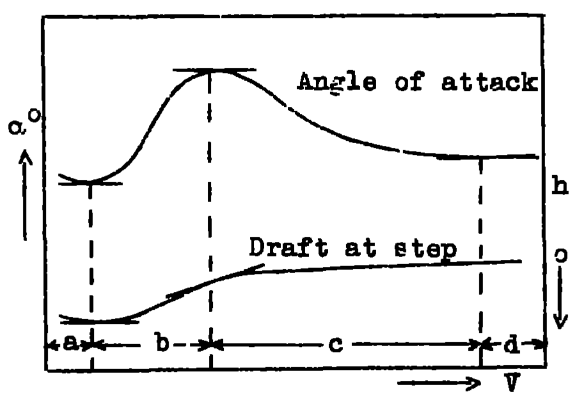
TABLE V

V	λ	$\frac{lg}{V^2}$	$\frac{2M}{\frac{\Delta}{2} l}$	$2M_h$	$2M_\Phi$	$2M_\Delta$	$2M_h - (2M_\Phi + 2M_\Delta)$
10	0.197	0.0982	5.2	6210	1085	898	4227
12	.298	.0646	4.14	4720	1040	854	3866
14	.45	.0475	3.2	3420	980	804	1636
16	.7	.0364	2.2	2190	900	748	542
20	1.45	.0214	1.08	883	754	612	-483
24	3.0	.0149	.526	311	579	444	-712
28	5.1	--	--	103.6	374	253	-513.4



Plowing F = 0 - 0.5
Transition F = 0.5 - 2.5
Planing F > 2.5

Figure 1



a, Plowing stage
F = 0 - 0.5
b, Transition stage
F to 0.5 to 2.5
c, Planing F to 10
d, Take-off F > 10

Figure 2

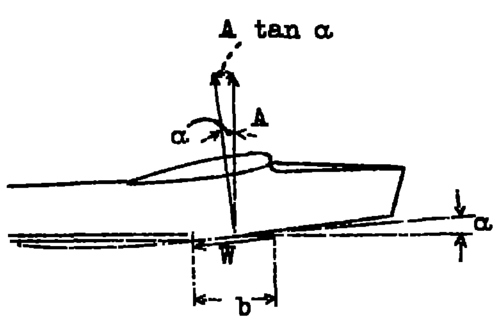


Figure 3

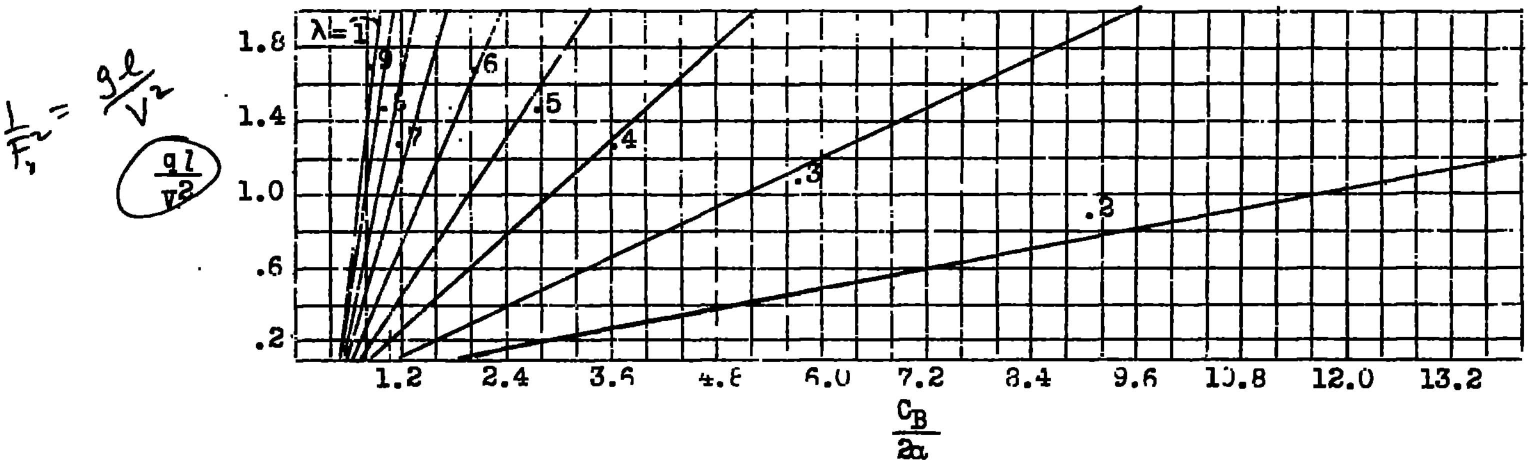


Figure 4

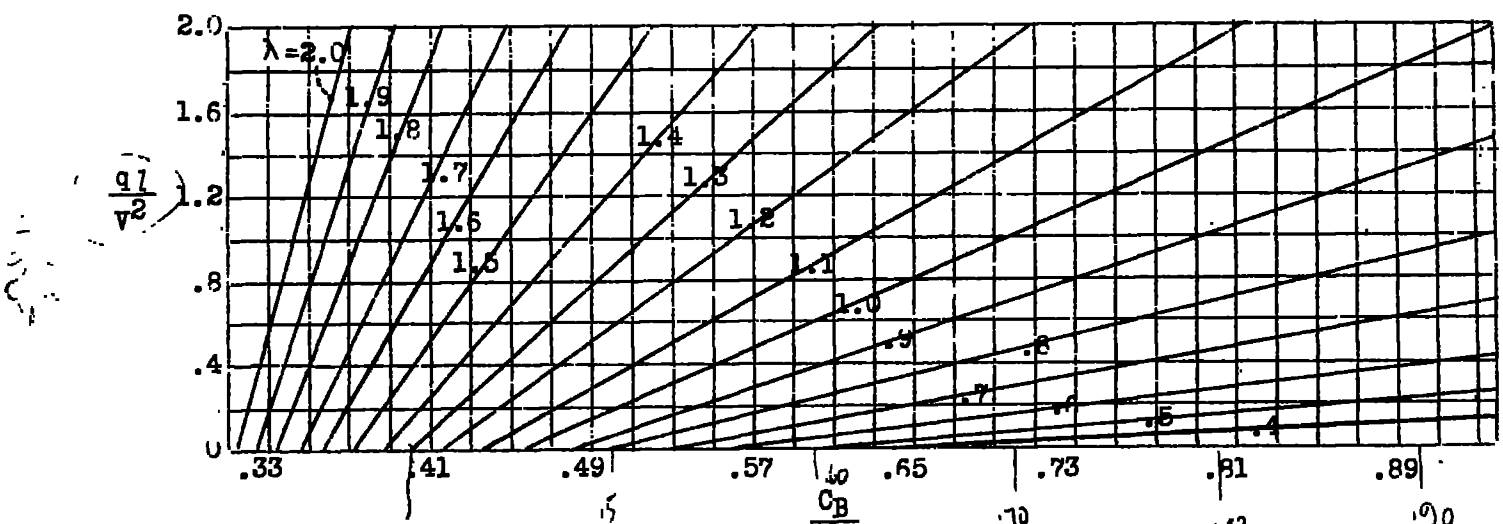


Figure 5

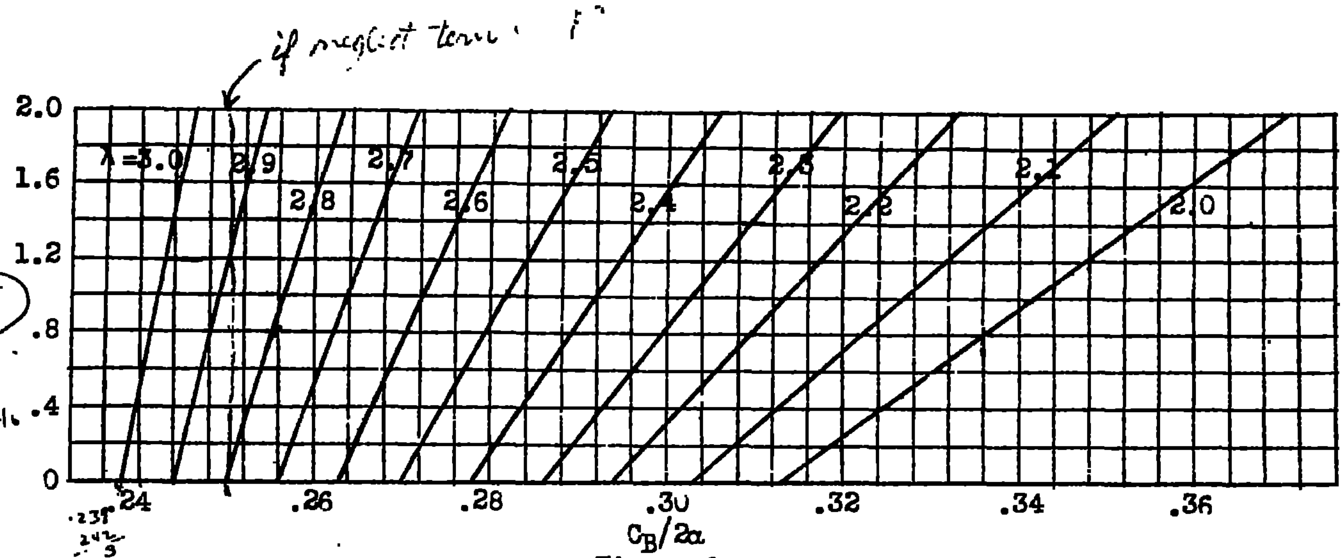


Figure 6

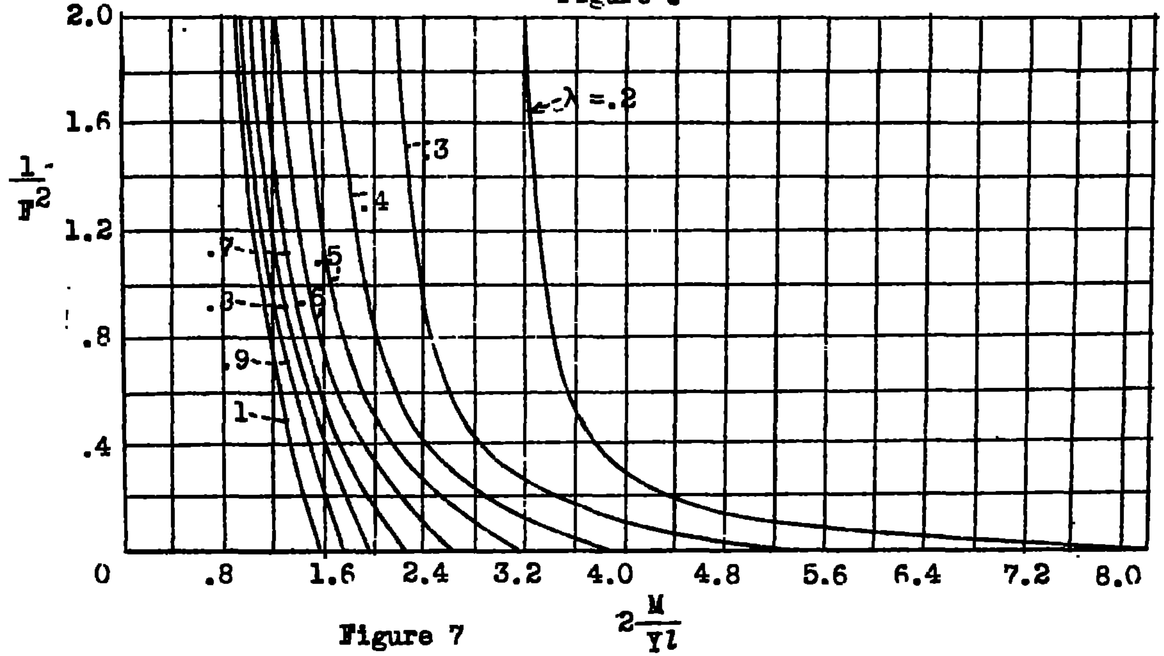


Figure 7

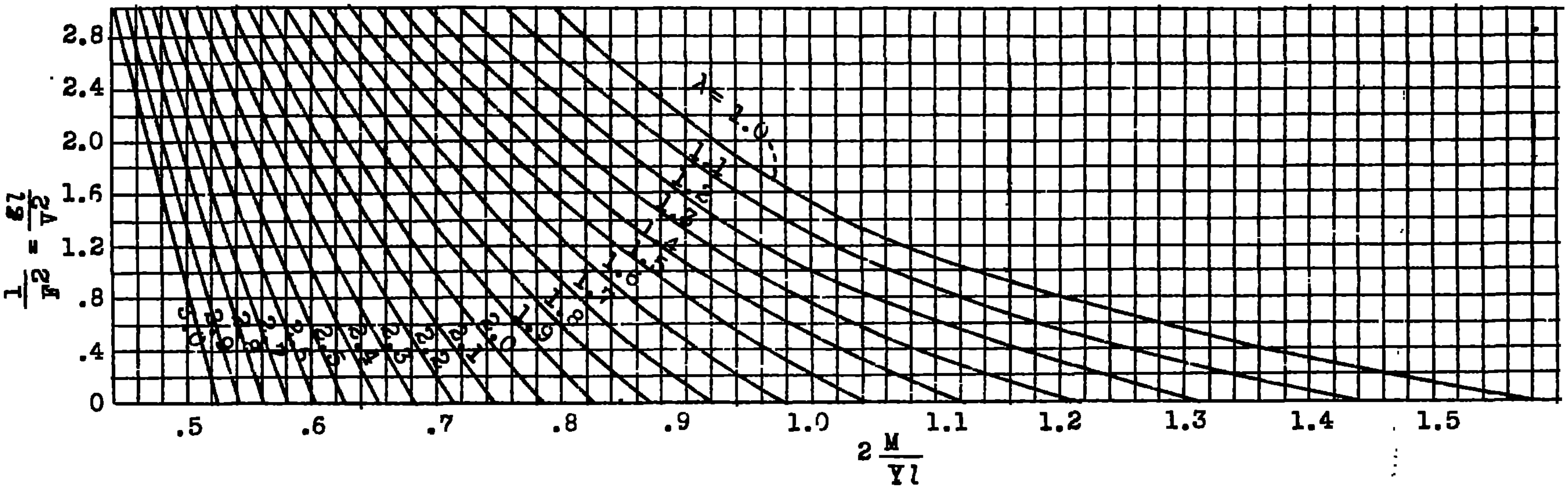


Figure 8

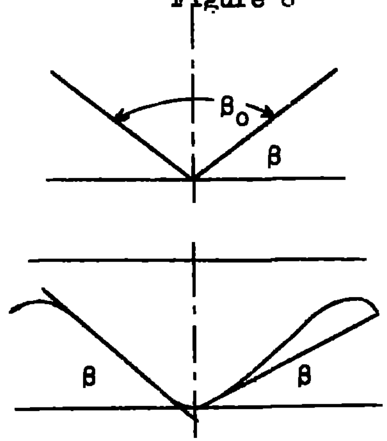


Figure 9.- Straight V and concave bottom plates.

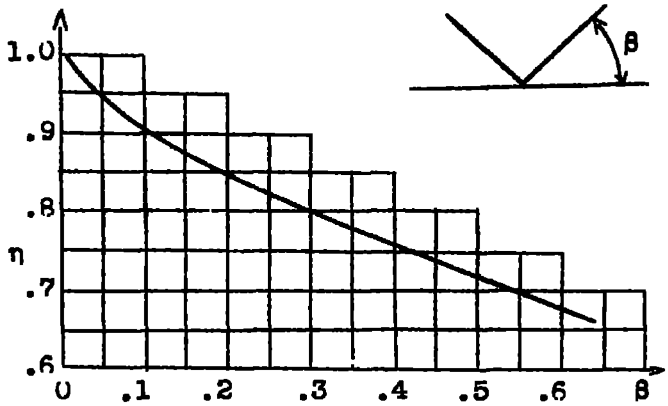


Figure 10

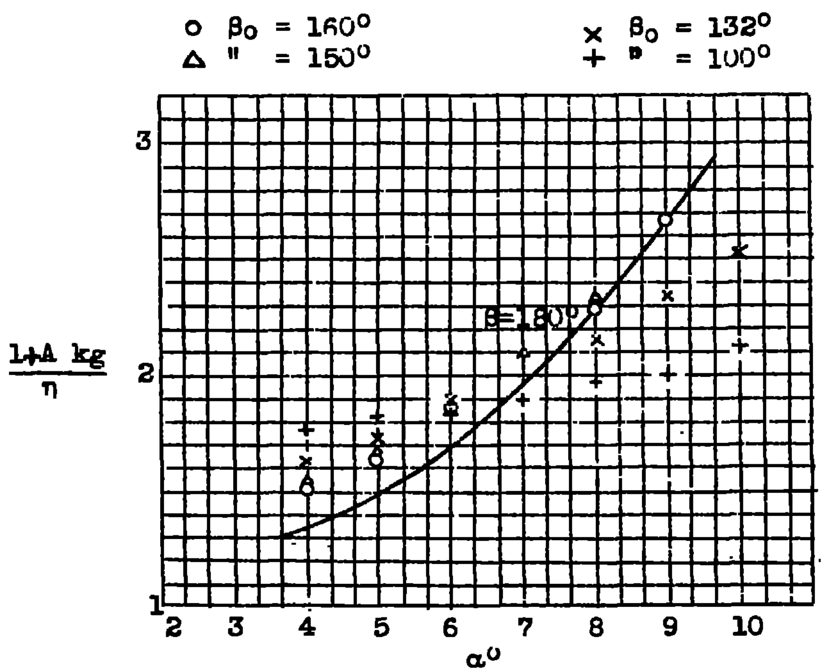


Figure 11

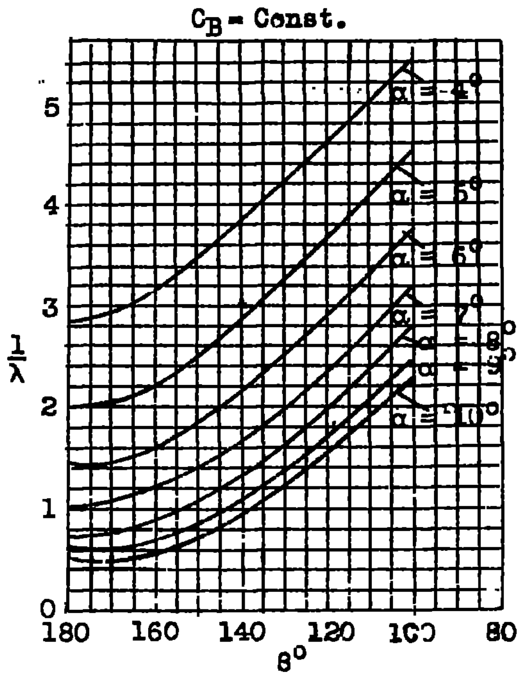


Figure 12.

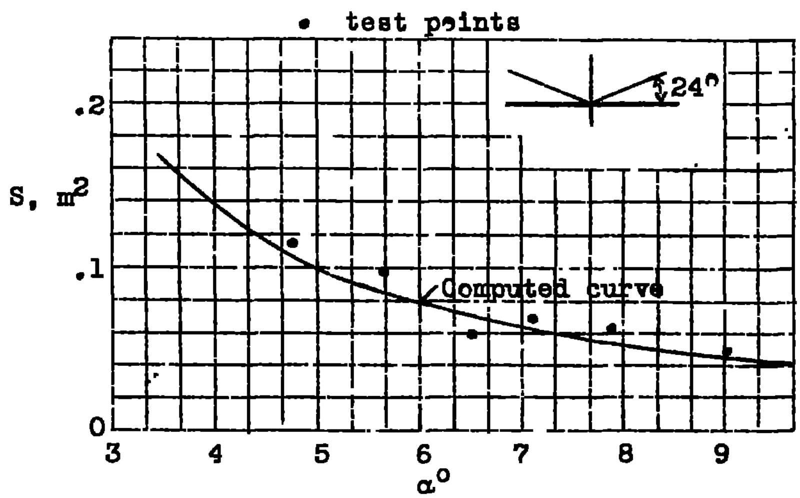


Figure 13.- Test conducted in CAHI tank. A = 16 kg., V = 7.35 meters per second.

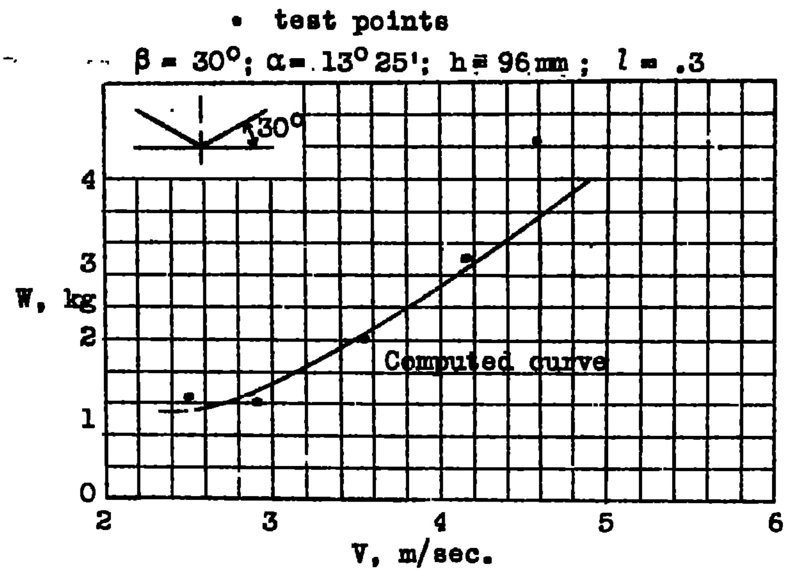


Figure 14.- Comparison of computed curve with test points obtained in the NIVK tank.

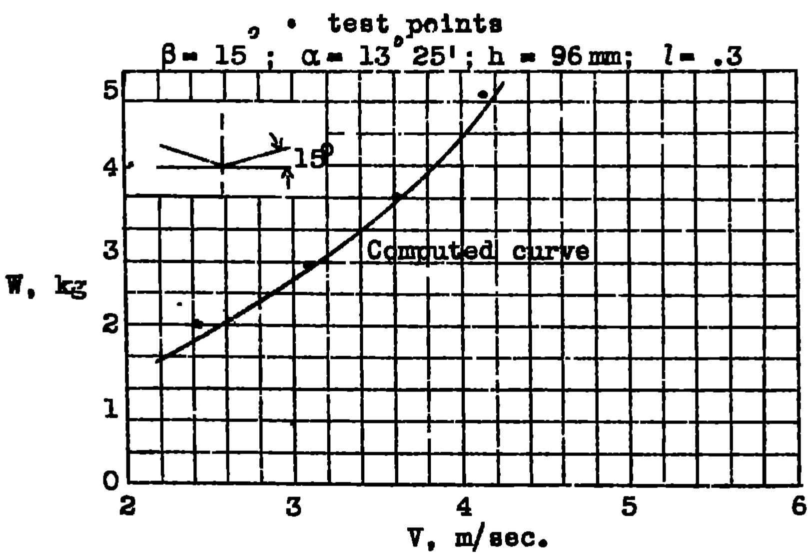


Figure 15.- Comparison of computed curve with test points obtained in the NIVK tank.

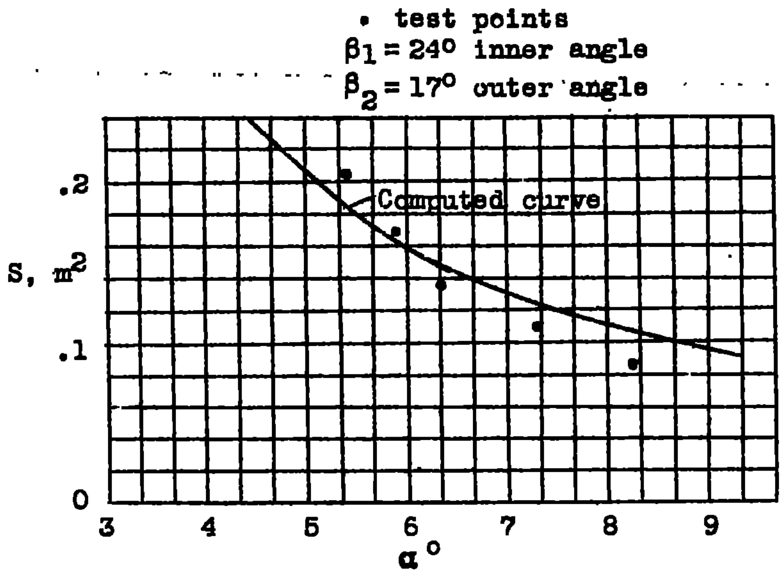


Figure 16.- Test conducted in CAHI tank. A = 18 kg., V = 6 meters per second.

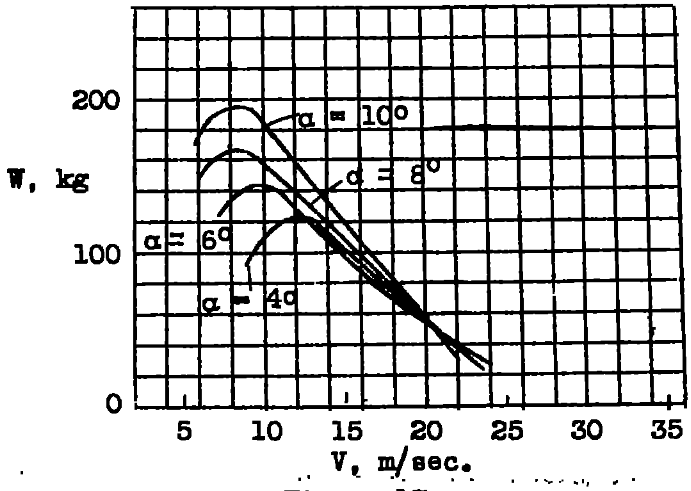


Figure 17.

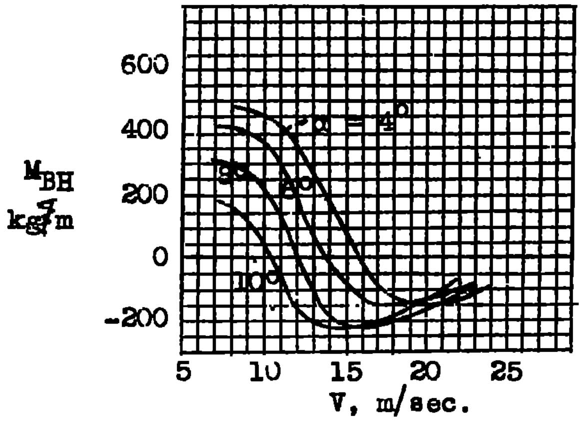


Figure 18

a, Experimental curve.
b, Computed resistance curve.
c, Experimental curve.
d, Computed trim angle curve.
e, Curve of elevator moments against speed.

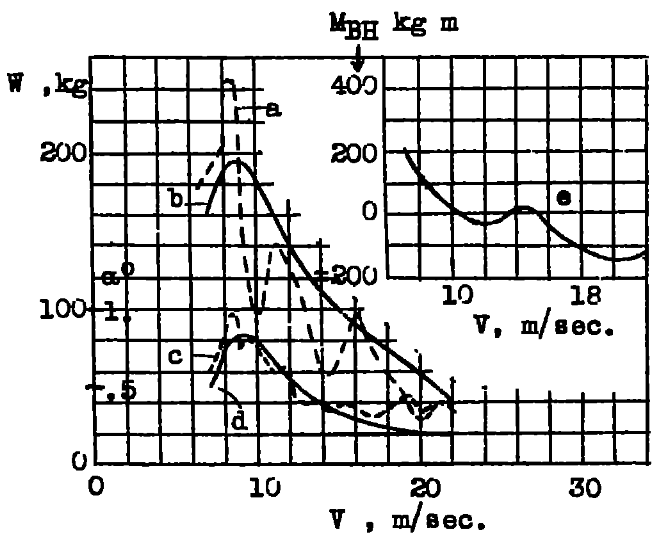


Figure 19

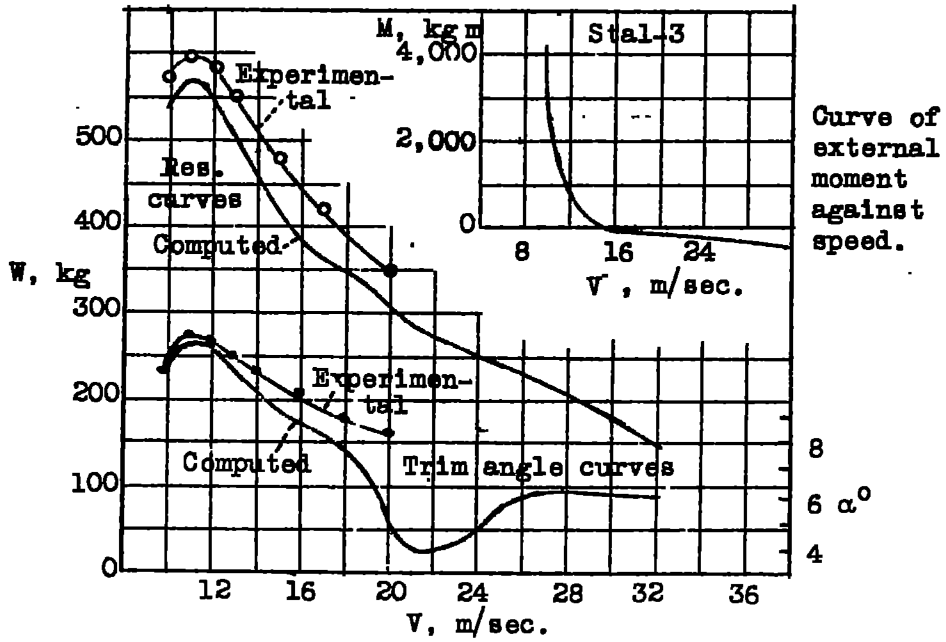


Figure 20

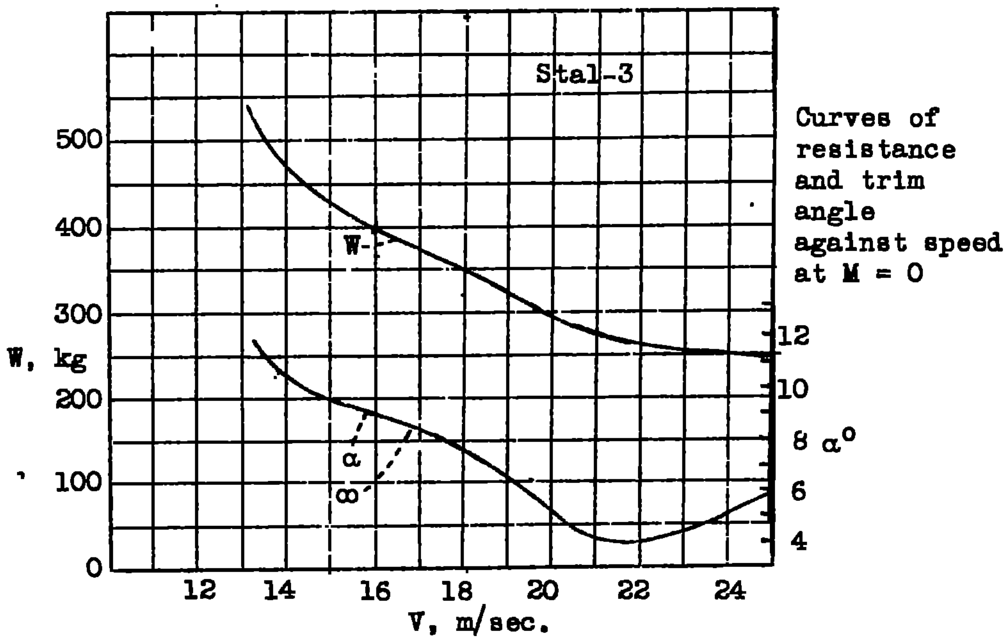


Figure 21

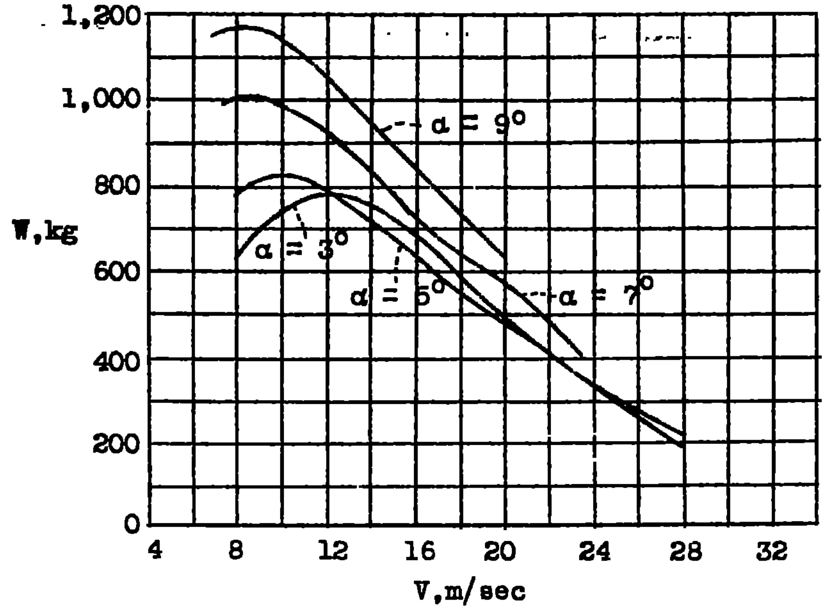


Figure 22.- Computed curves of resistance against velocity for the model N.A.C.A. 11 hull.

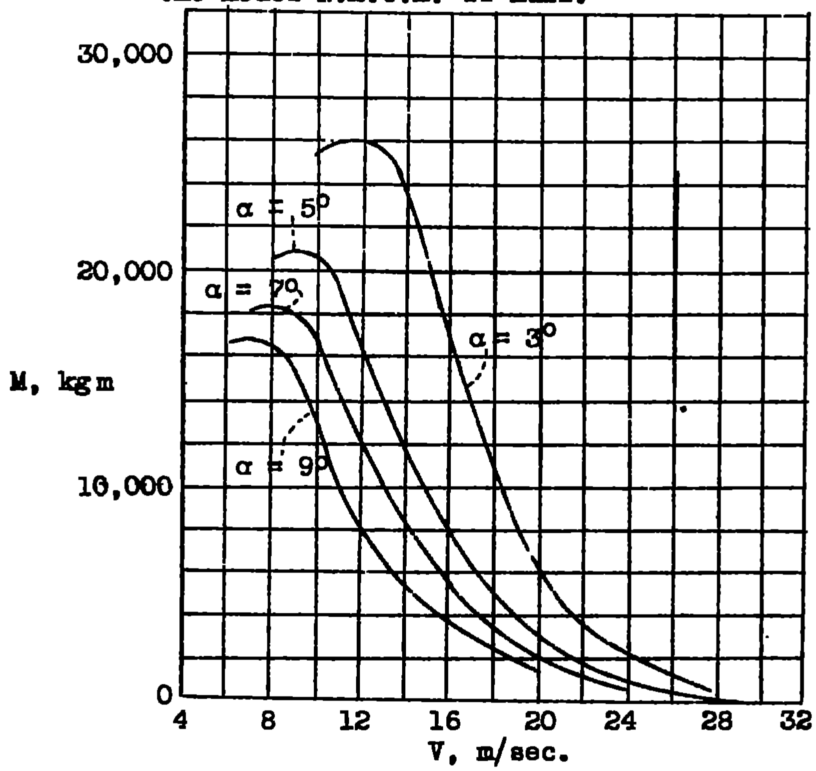


Figure 23.- Computed curves of moments in get-away for the model N.A.C.A. 11 flying-boat hull.

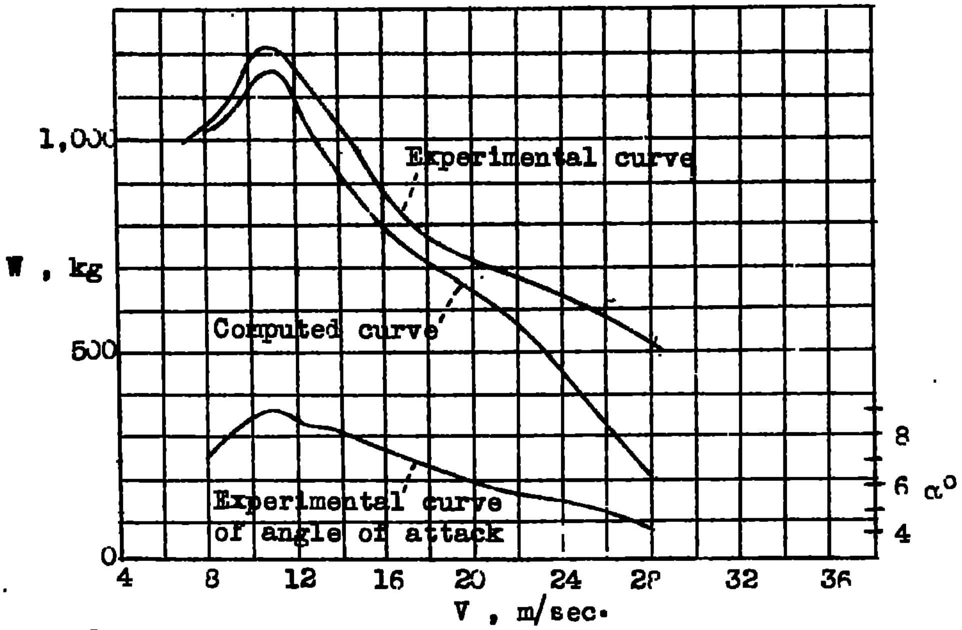


Figure 24.- Curves of resistance and angle of attack in get-away model N.A.C.A.-11 hull.

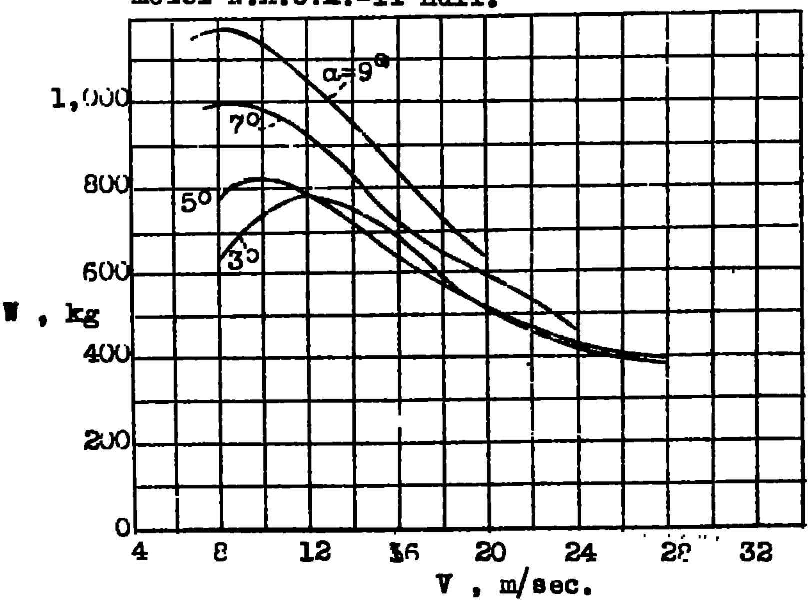


Figure 25.- Computed curves of resistance in get-away for the model N.A.C.A.-11 hull.

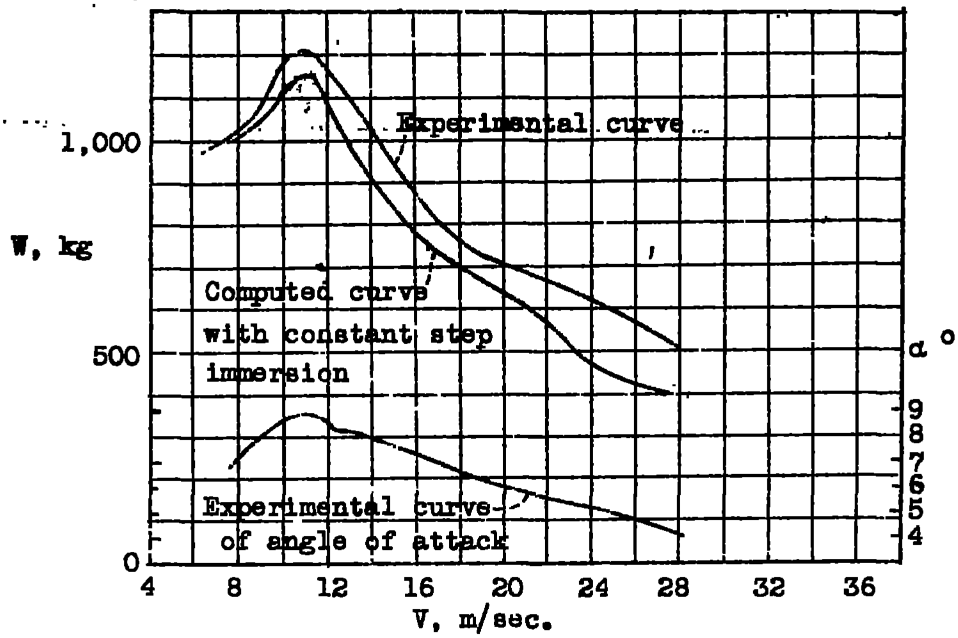


Figure 26.- Curves of resistance and angles of attack in get-away for the model N.A.C.A. 11 hull.

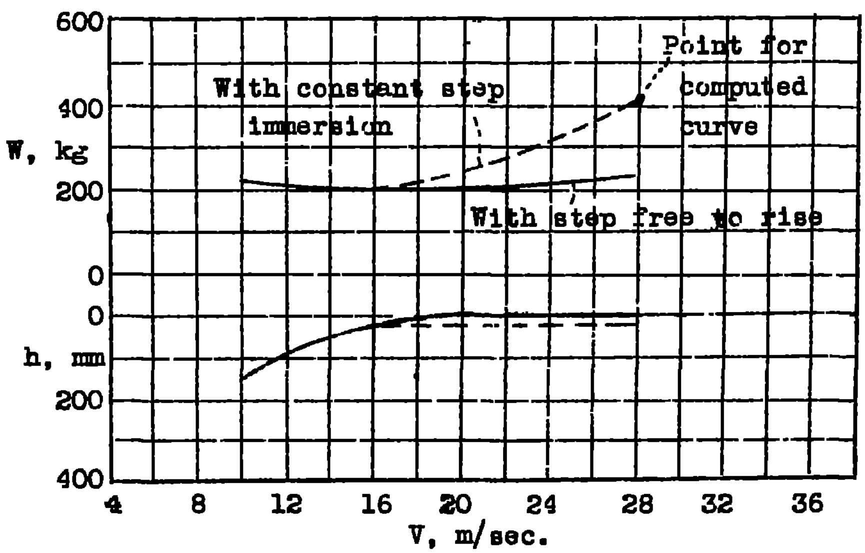


Figure 27.- Computed curve of resistance of flying-boat hull N.A.C.A. 11 at $\alpha = 5^{0}$ (load A = 1,445 kg)

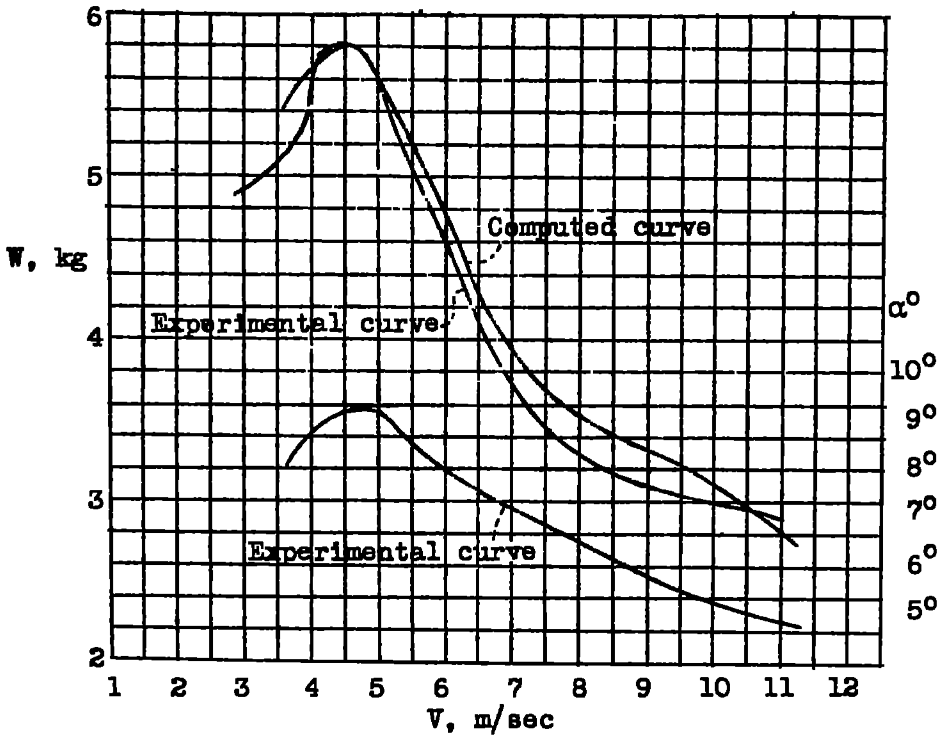


Figure 28.- Computed and experimental resistance curves against speed for the model N.A.C.A. 11 hull.